Workshop Cargèse
Trends in physical and numerical modeling of multiphase flows,
CFD and its experimental validation for multiphase flows
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Lagrange-remap solver and low-diffusive interface capturing for air-water flows

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Outline

1. System of interest
2. Numerical method
3. Antidiffusive procedure for the interface
4. Numerical experiments & validation
5. Concluding remarks

System of interest

- Air-water multifluid flow
- Viscous effects neglected
- Surface tension neglected

- Transport equation to track the mass gas fraction transport c_g

- « Three-equation » volume averaged model (one velocity) with pressure equilibrium closure

Scope, applications, project

- Wave breaking/wave impact analysis
- Pressure impact under sloshing conditions
- Pipe flows

- … a milestone before including more Physics (viscosity, surface tension, …)
- … prototype multiphase code solver suitable for GPU parallel computing.

System of equations

- Isentropic Euler system for gas
- Isentropic Euler system for (compressible) liquid
- + interface (kinematic/dynamic) conditions

- Trick : make use of a colored fluid function

$$\partial_t z + \boldsymbol{u} \cdot \nabla z = 0, \quad z \in \{0, 1\}.$$

- Numerical approach: consider

$$\partial_t z + \boldsymbol{u} \cdot \nabla z = 0, \quad z \in [0, 1]$$

- Use the mass gas fraction for example as color function:

$$\partial_t c_g + \boldsymbol{u} \cdot \nabla c_g = 0, \quad c_g = \frac{\alpha \rho_g}{\rho}$$

$$\alpha = \text{ gas void fraction}, \quad \rho = \alpha \rho_g + (1 - \alpha)\rho_\ell.$$

- We get mass conservation per phase :

$$\partial_t(\alpha \rho_g) + \nabla \cdot (\alpha \rho_g \boldsymbol{u}) = 0,$$

$$\partial_t((1 - \alpha)\rho_\ell) + \nabla \cdot ((1 - \alpha)\rho_\ell \boldsymbol{u}) = 0,$$

- + momentum equation

$$\partial_t(\rho \boldsymbol{u}) + \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u}) + \nabla p = \rho \boldsymbol{g}$$

- + closure : pressure equilibrium closure:

$$p = p_g(\rho_g) = p_\ell(\rho_\ell) \longrightarrow \varphi(\alpha) = 0.$$

Choice of the EOS for each fluid

- For the gas phase, we use a *perfect gas law* :

$$p_g(\rho_g) = p^0 \left(\frac{\rho_g}{\rho_g^0}\right)^{\gamma_g}, \quad \gamma_g = 1.4,\ \rho_g^0 = 1.28\ kg/m^3,\ p^0 = 10^5\ Pa.$$

- For the water : we use a *modified Tait equation* for the liquid

$$p_\ell(\rho_\ell) = p^0\ \left\{1 + K\left[\left(\frac{\rho_\ell}{\rho_\ell^0}\right)^{\gamma_\ell} - 1\right]\right\}, \quad K = \frac{\rho_\ell^0 c_s^2}{p^0 \gamma_\ell}$$

with for instance: $\gamma_l = 3.5$, $\rho_l^0 = 1000$, $c_s = 350$ m.s^{-1}, ($K = 350$)

NB: weakly compressible: if $p = \dfrac{p^0}{2}$, then $\rho_\ell = 999.59$

Pressure equilibrium equation

- From the knowledge of the conservative variables

$W_g = \alpha\rho_g$ and $W_g = (1-\alpha)\rho_g$, we have to solve :

$$p_g(\rho_g) = p_\ell(\rho_\ell)$$

i.e.

$$\varphi(\alpha) = \left(\frac{W_g}{\alpha\rho_g^0}\right)^{\gamma_g} - 1 - K\left[\left(\frac{W_\ell}{(1-\alpha)\rho_\ell^0}\right)^{\gamma_\ell} - 1\right] = 0, \quad \alpha \in]0,1[.$$

- NB : very stiff function, the choice of the iterative solver requires attention (initial guess, surrogates, Newton, etc.)

- In fact, there is a trick … :

Trick for initial guess :

- The liquid mass conservation equation can be written in the form :

$$\partial_t(1-\alpha) + \nabla \cdot [(1-\alpha)\boldsymbol{u}] = -\frac{D_t \rho_\ell}{\rho_\ell}.$$

- Under the *weak compressibility assumption* or the liquid phase, we get

$$\partial_t \alpha + \nabla \cdot (\alpha \boldsymbol{u}) = 0.$$

- A numerical scheme is applied to this additional ``guess equation'' to compute initial guesses of the iterative solver

  $\rightarrow$ Newton algorithm converges in 2 iterates with acceptable accuracy (strong improvement in CPU time).

Numerical scheme

Lagrange-remap strategy

- First write the equations in Lagrangian form

$$\frac{d}{dt}\int_{\Omega_t} \alpha\rho_g \, dx = 0,$$

$$\frac{d}{dt}\int_{\Omega_t} (1-\alpha)\rho_\ell \, dx = 0,$$

$$\frac{d}{dt}\int_{\Omega_t} \rho \boldsymbol{u} \, dx + \int_{\Omega_t} \nabla p \, dx = \int_{\Omega_t} \rho \boldsymbol{g}.$$

(Lagrange integral form here)

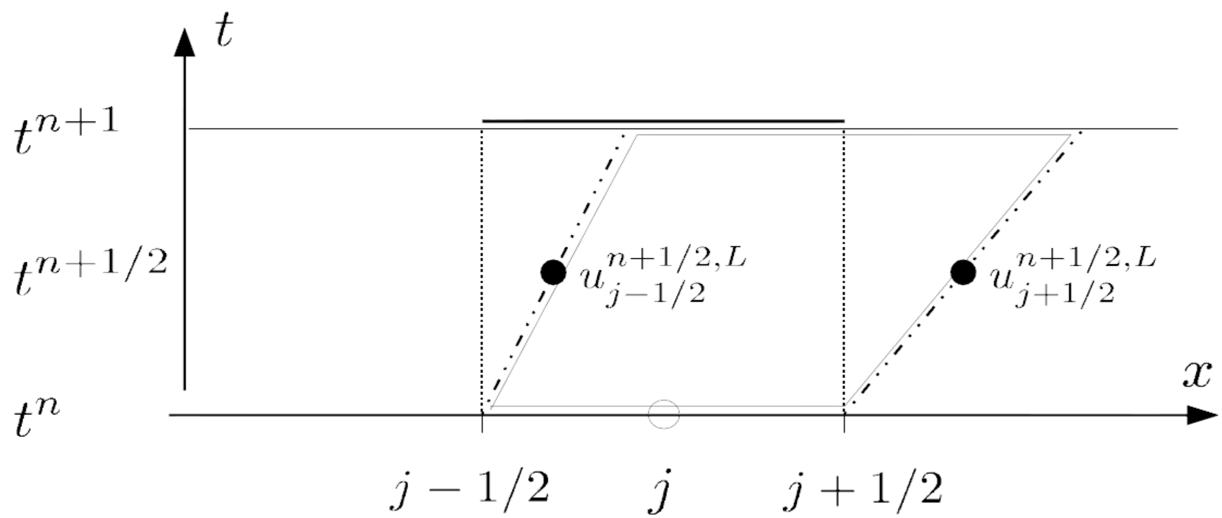
- Integrate them in time over a time step
- Project the quantities on the fixed Eulerian grid (remap)

Conservative form

- Lagrange-remap schemes can be rewritten actually in conservative form [De Vuyst et al, CRAS Mécanique 2012].
- 1D case :

$$(\alpha\rho_g)_j^{n+1} = (\alpha\rho_g)_j^{n+1} - \frac{\Delta t}{h}\left[(\Phi_{m,g})_{j+1/2}^{n,n+1} - (\Phi_{m,g})_{j-1/2}^{n,n+1}\right],$$

$$(\Phi_{m,g})_{j+1/2}^{n,n+1} = \alpha_{j+1/2}^{n+1,L}\,(\rho_g)_{j+1/2}^{n+1,L}\,u_{j+1/2}^{n+1/2,L}$$



Antidiffusive strategy

$$(\alpha\rho_g)_j^{n+1} = (\alpha\rho_g)_j^{n+1} - \frac{\Delta t}{h}\left[(\Phi_{m,g})_{j+1/2}^{n,n+1} - (\Phi_{m,g})_{j-1/2}^{n,n+1}\right],$$

$$(\Phi_{m,g})_{j+1/2}^{n,n+1} = \alpha_{j+1/2}^{n+1,L}\,(\rho_g)_{j+1/2}^{n+1,L}\,u_{j+1/2}^{n+1/2,L}$$

Void fraction at interface : how to compute it ?

- Strategy : « be as sharpest as possible for step-shaped color functions while staying stable (just at the limit) »
- Idea : design a combination of upwinding scheme and downwind scheme [Lagoutière Després 2002, Kokh-Lagoutière 2010]
- The advection process can be seen as a over-compressive « limiting » procedure (superbee-like, hyperbee, ...)

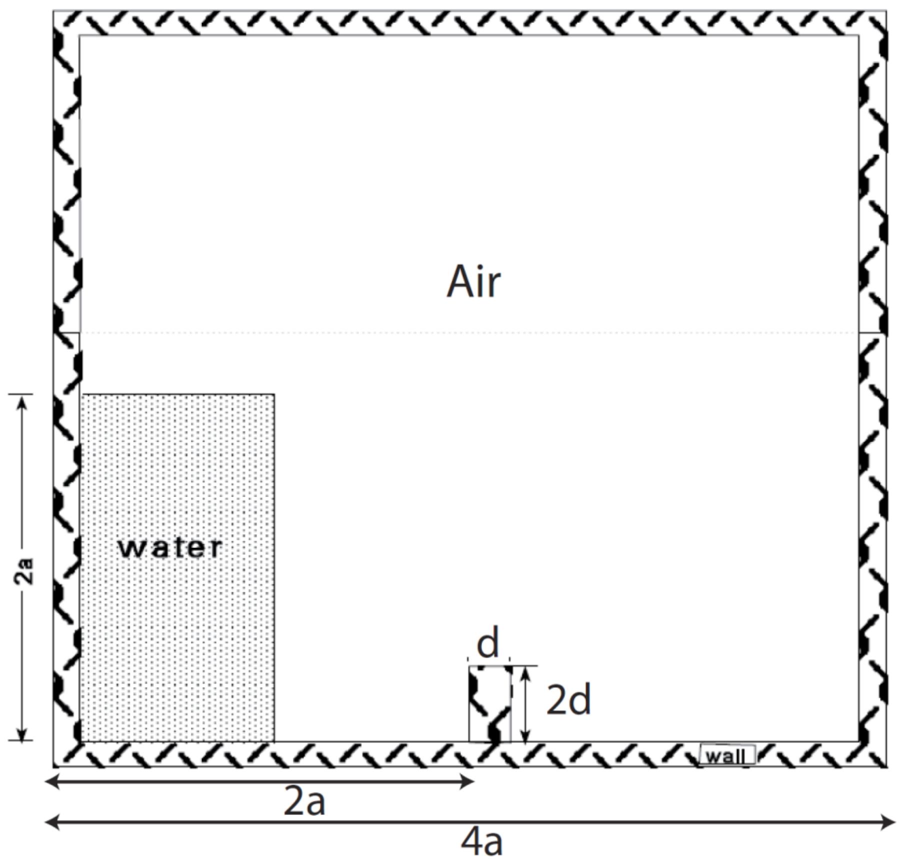
Test cases

and numerical experiments

(+ comparison to physical experiments for some of them)

1. Collapse of a liquid column with an obstacle

O. Ubbink Numerical prediction of two-fluid systems with sharp interfaces, PhD thesis (1997)



$$a = 0.146 \text{ m},\ d = 0.024 \text{ m}$$

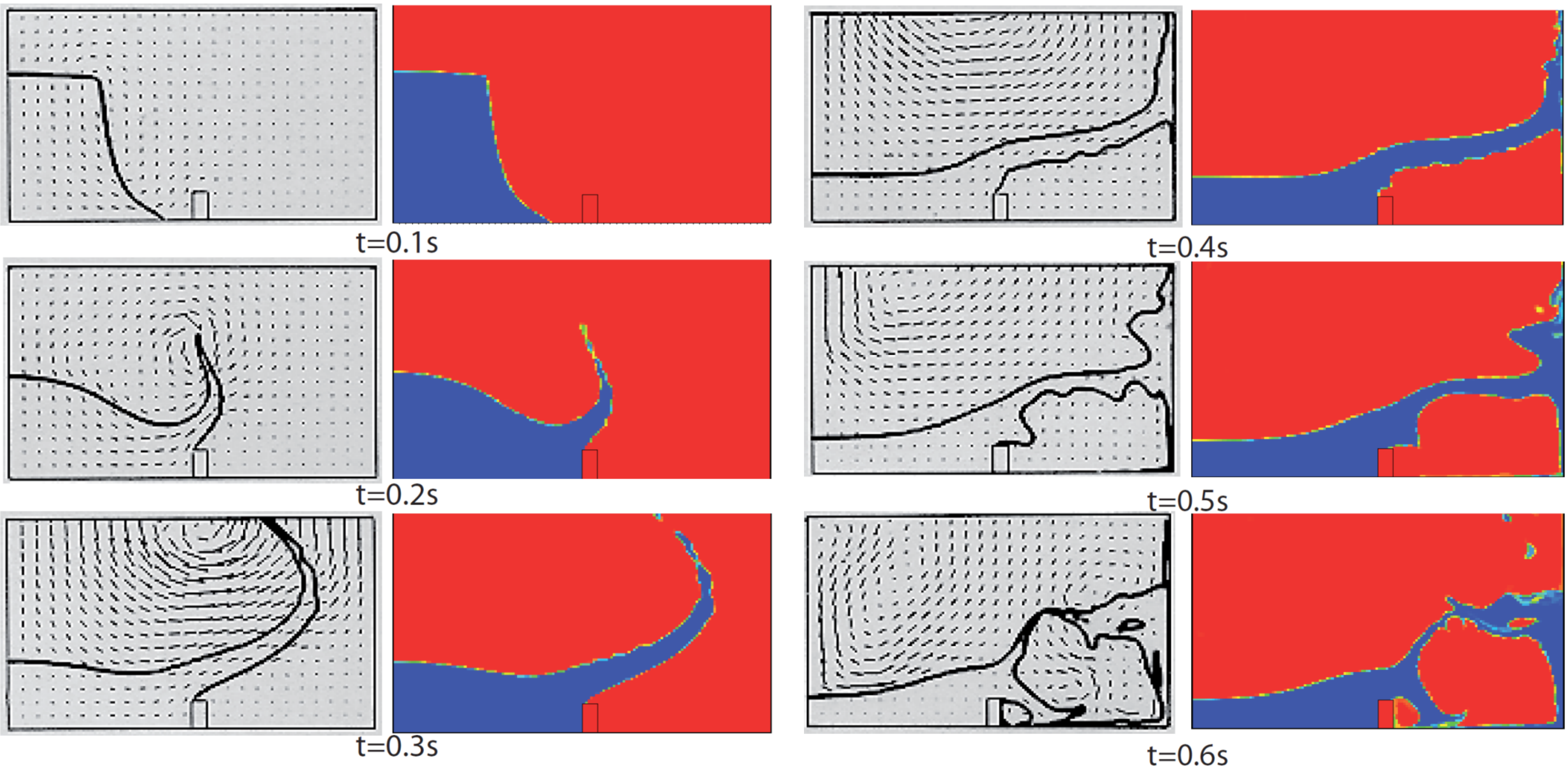
$$Nx = Ny = 150$$

$$\gamma_g = 1.4,\ \gamma_l = 7$$

$$\rho_0^g = 1.28 \text{ kg.m}^{-3},\ \rho_0^l = 1000 \text{ kg.m}^{-3}$$

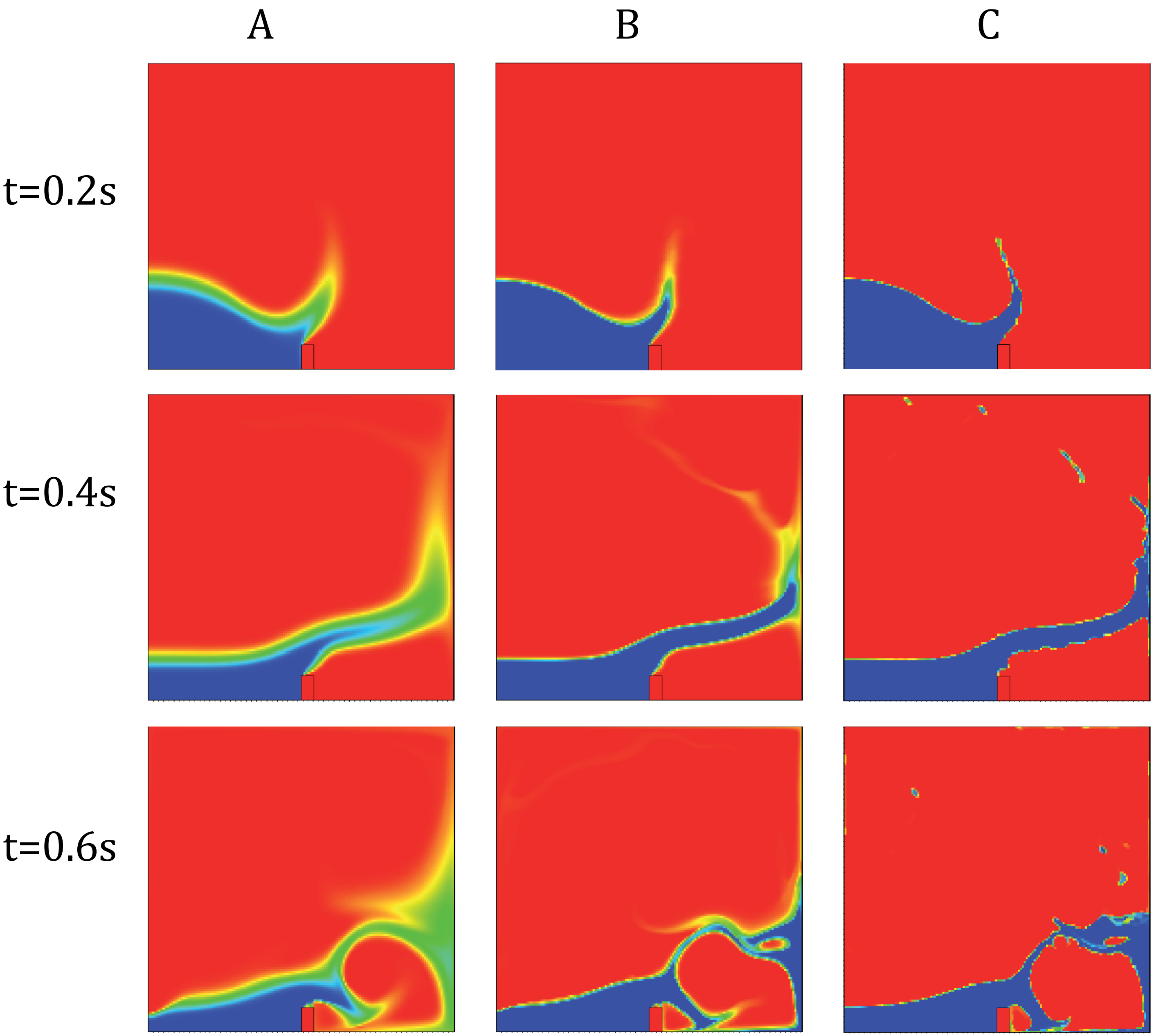
$$P_0 = 10^5,\ c_{\text{sound}} = 350 \text{ m.s}^{-1}$$

Collapse with an obstacle – comparison with incompressible fluid model



Num. Result. Ubbink | Num. Result. with Odyssey | Num. Result. Ubbink | Num. Result. with Odyssey

Benefits of the antidiffusive approach



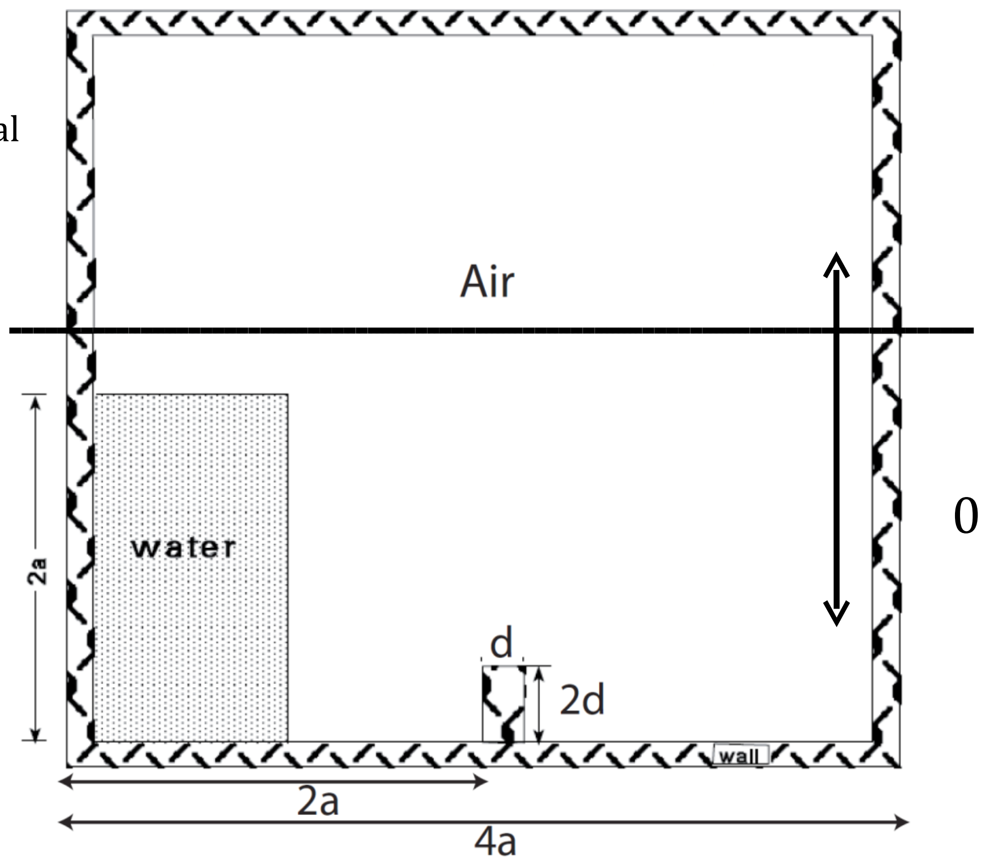
Evolution of ✓□for different remapping algorithms

- ✓Case A: 1^{st} order with upwind
- ✓Case B: 2^{nd} order with upwind
- ✓Case C: 1^{st} order with Low-Diff.

- Another case of collapse of a liquid column with an obstacle

D. M. Greeves Simulation of viscous water column collapse using adapting hierarchical grids J. Num. Meth. Fluids 2006

End of the experimental open tank



0.6 m

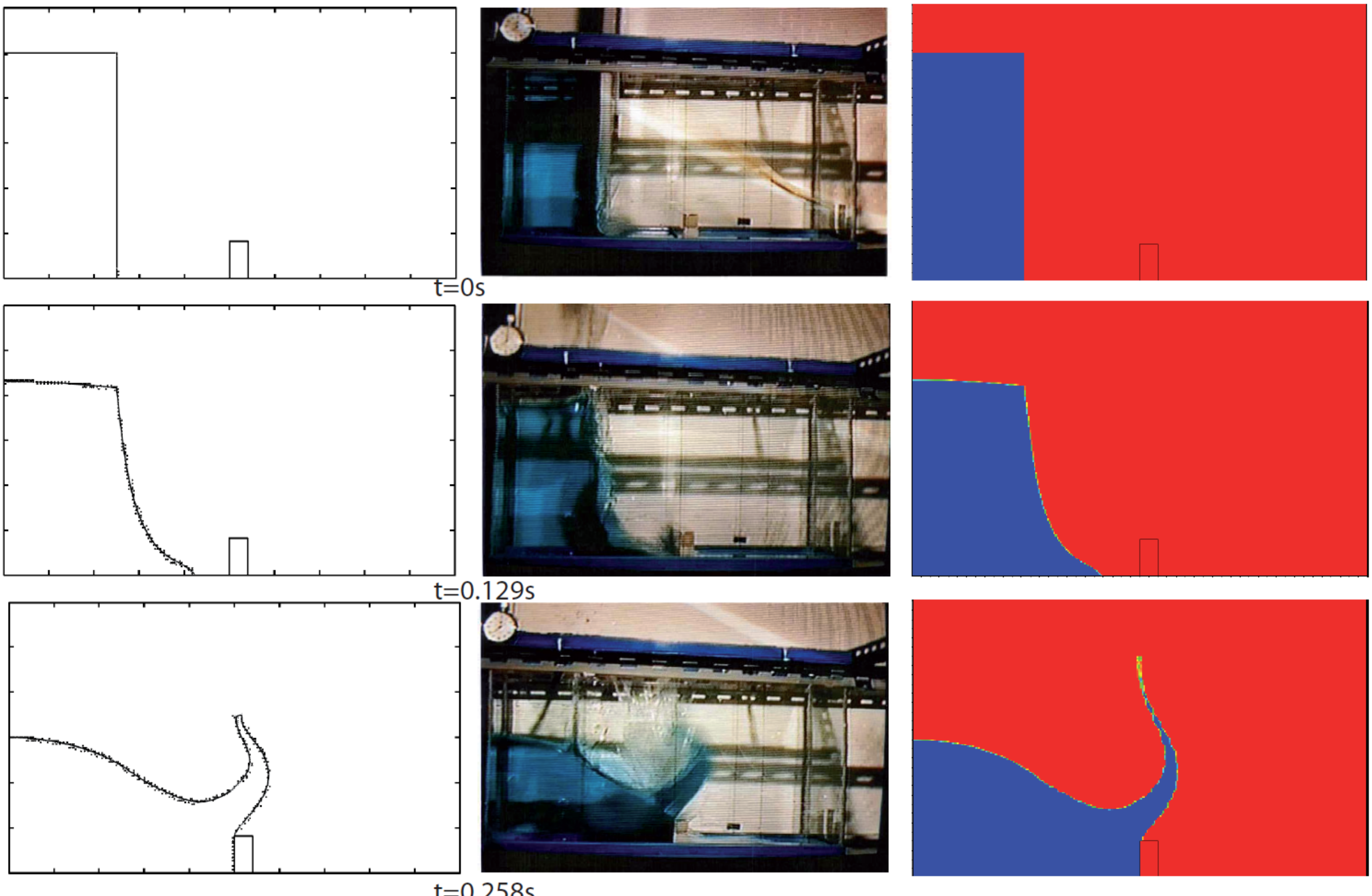
$$a = 0.25,\ d = 0.04$$
$$Lx = Ly = 1 \text{ m}$$

$$Nx = Ny = 300$$
$$\gamma_g = 1.4,\ \gamma_l = 7$$
$$\rho_0^g = 1.\ \text{kg.m}^{-3},\ \rho_0^l = 1000\ \text{kg.m}^{-3}$$
$$P_0 = 10^5,\ c_{sound} = 350\ \text{m.s}^{-1}$$

Exp: from

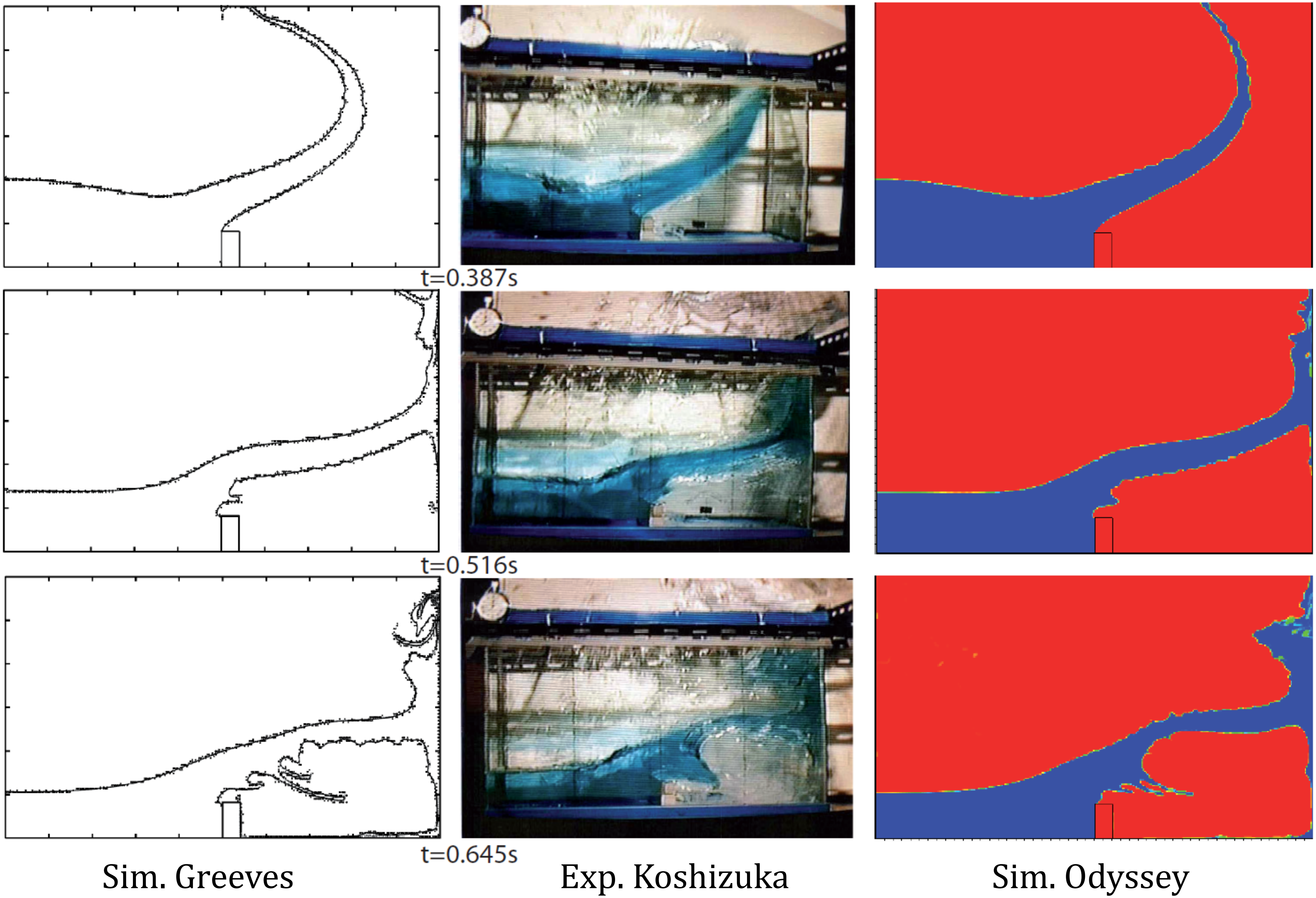
Koshizuka et al A particle method.. Comp. Fluid Mech. 1995

Simulations of a dam break



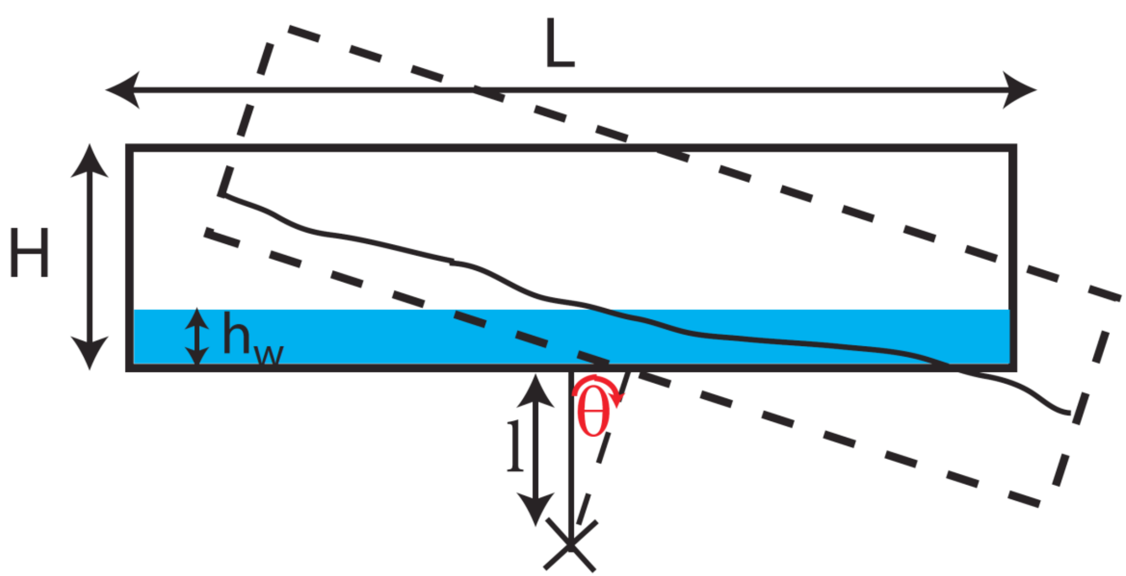
Sim. Greeves Exp. Koshizuka Sim. Odyssey

Simulations of a dam break



Sloshing test cases – pitch motion

- Sloshing due to the **pitch motion** of a rectangular tank:



J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012

$$L = 0.64m,\ H = 0.14m,\ h_w = 0.03m$$

$$Nx = 300,\ Ny = 67$$

The tank is oscillating as a **pendulum** according to:

$$\theta(t) = \theta_0 \sin(\omega_r t)$$

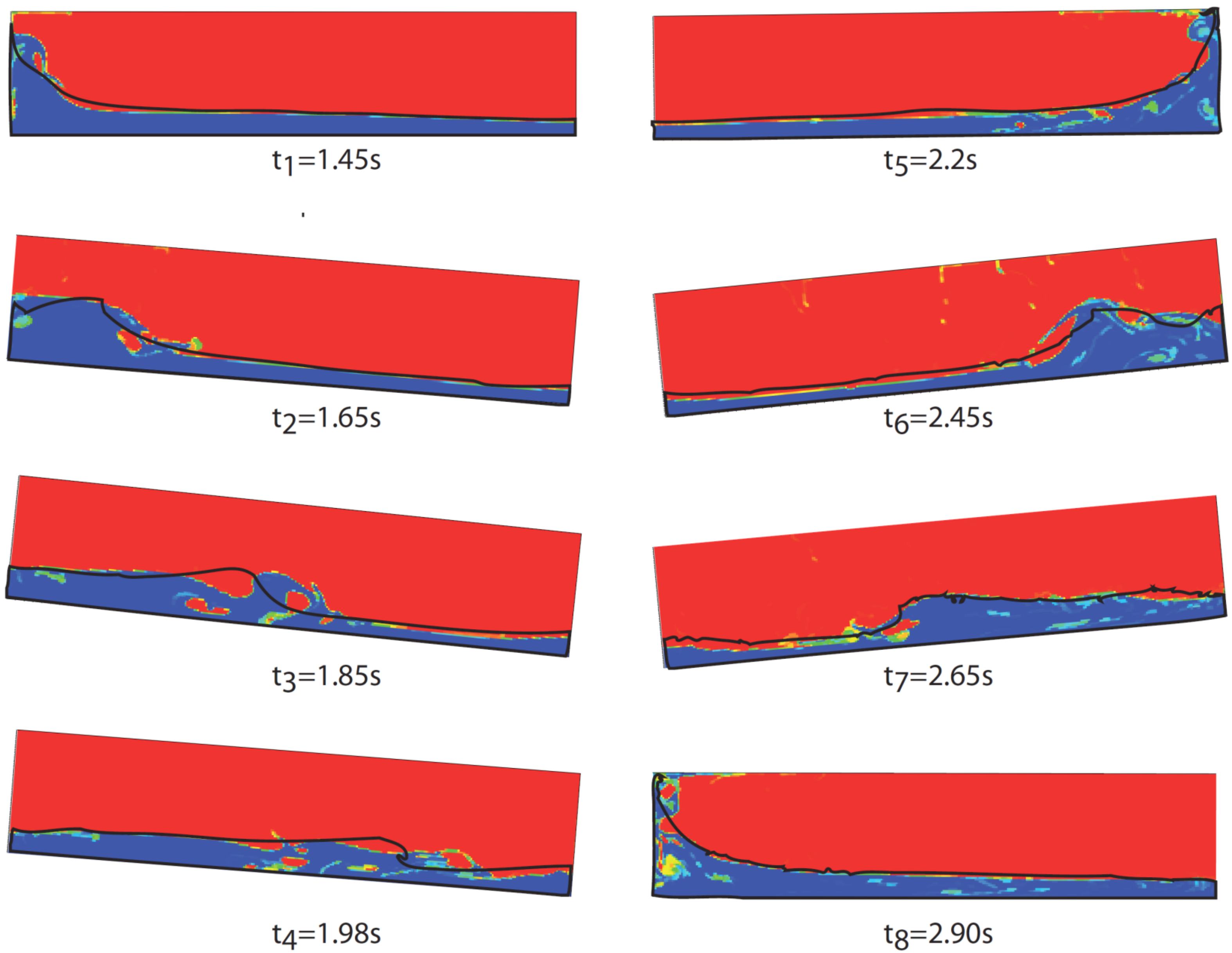
with $\theta_0 = 6^\circ,\ \omega_r = 4.34\ \text{rad/s}\ (T = 1.45\ \text{s})$

Simulation are performed in the frame of reference of the tank.

Comparison SPH – present method

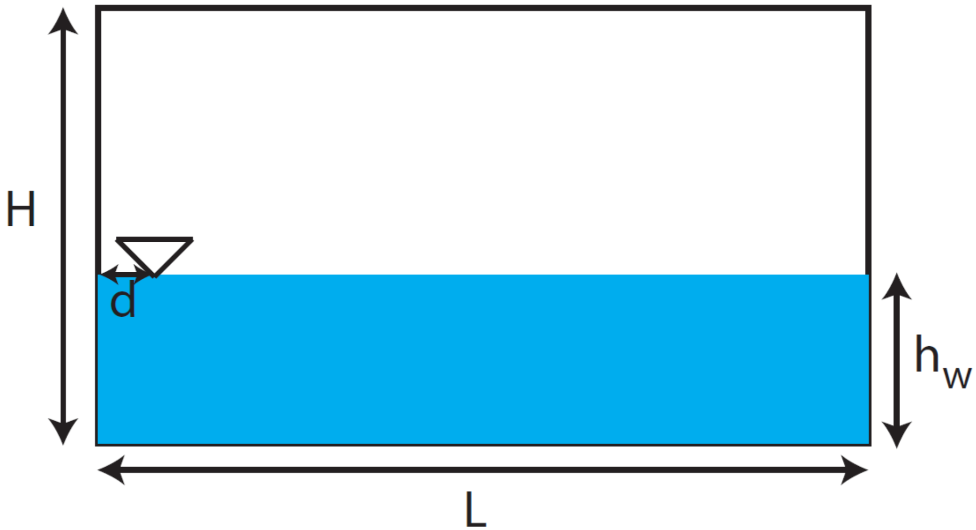
We superimpose the profile of the article:

J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012



Sloshing test cases – surge motion

- Sloshing due to the **surge motion** of a rectangular tank:



J.R. Shao *et al* . An improved SPH method for modeling liquid sloshing dynamics. Comp. Fluids 2012

$L = 1.73m,\ H = 1.15m,\ h_w = 0.6m$

$d = 0.05$ m

$Nx = 173,\ Ny = 115$

The tank is moving horizontally according to:

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right)$$

with $A = 0.032$ m, $T = 1.3$ s $(\omega_{forced} = 4.83$ rad/s).

First natural frequency of the fluid in the box

$$\omega_{fluid} = \sqrt{g\frac{\pi}{L}\tanh(\frac{\pi}{L}h_w)} \approx 3.77 \text{ rad/s}$$

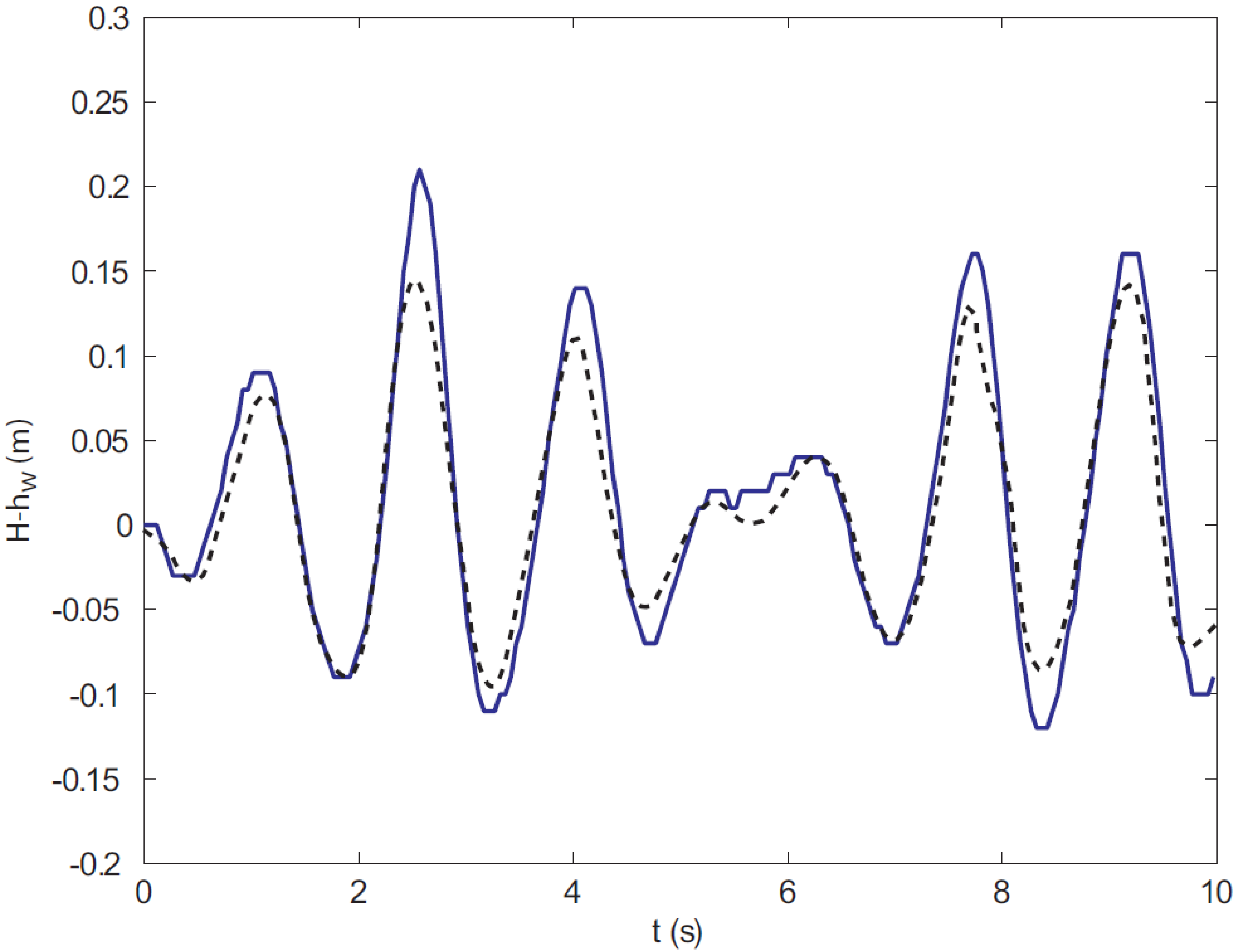
Two frequencies are acting ω_{fluid} and ω_{forced}

Experimentals results are available:

O.M. Faltinsen *et al* . Multidimensional modal analysis... J. Fluid. Mechanics 2000

Sloshing test cases – surge motion

Free surface elevation of water at the probe



with our code

Scanned experimental results of

O.M. Faltinsen *et al* . Multidimensional modal analysis... J. Fluid. Mechanics 2000

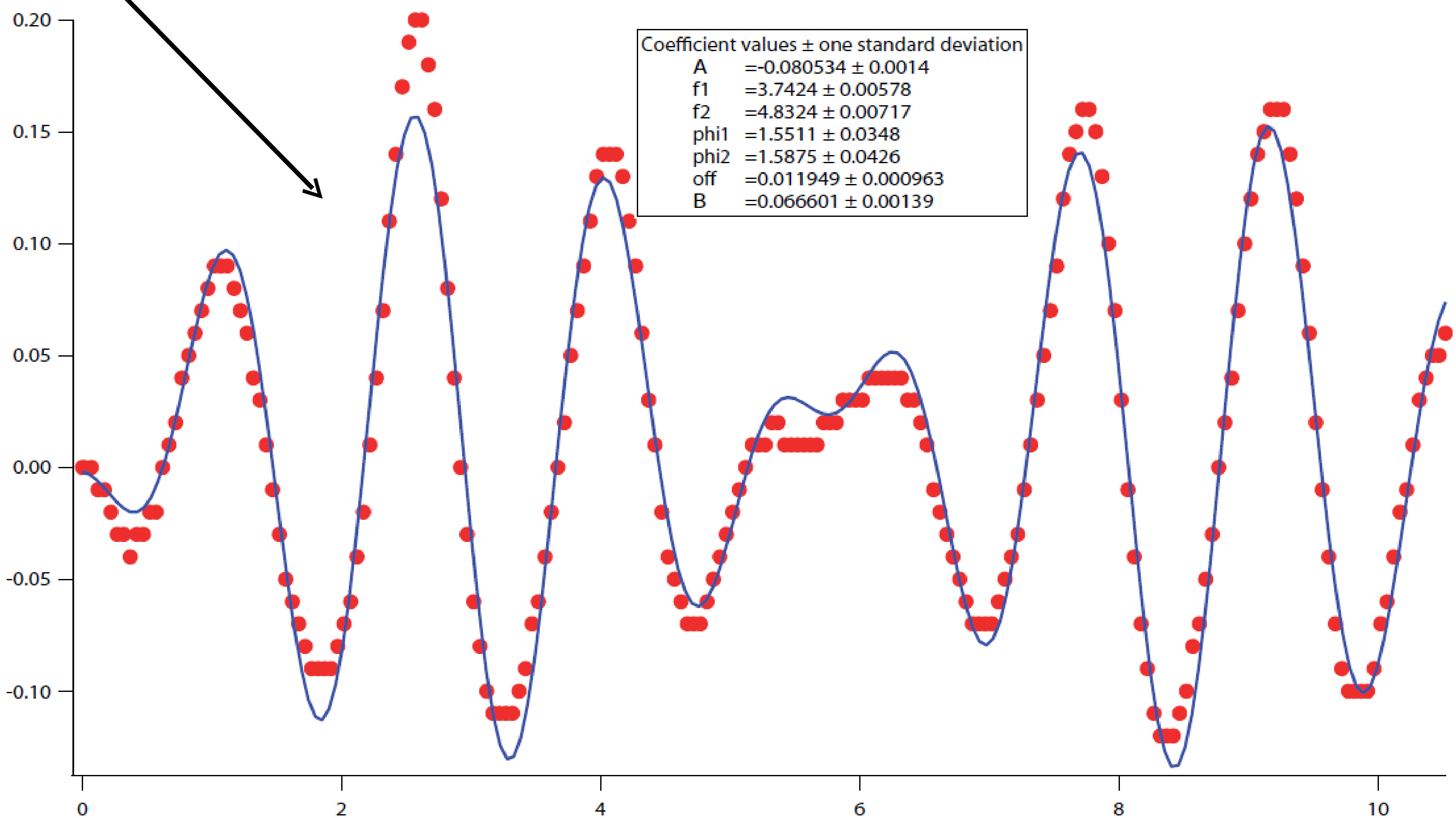
Sloshing test cases – surge motion

We search to **find a fit of** our curve with a function as a superposition of two signals

$$f(t) = A_1 \sin(f_1 t + \varphi_1) + A_2 \sin(f_2 t + \varphi_2)$$

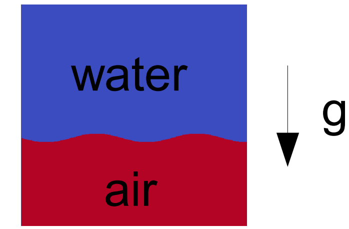
we get: $f_1 = 3.74 \pm 0.01$ rad/s, $f_2 = 4.83 \pm 0.01$ rad/s

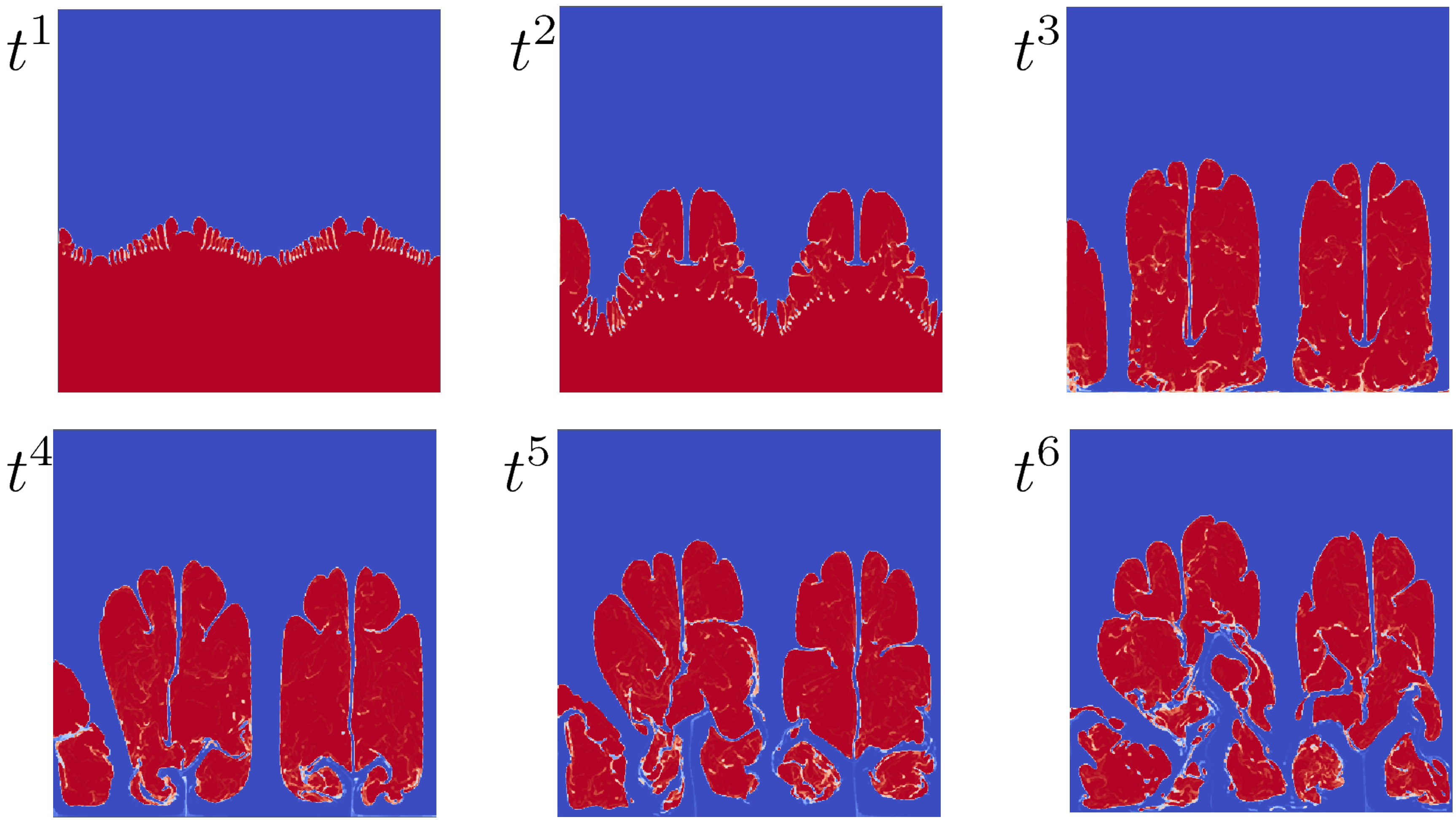
very close to $\omega_{\text{fluid}} \approx 3.77$ rad/s and $\omega_{forced} = 4.83$ rad/s



LT air-water Rayleigh-Taylor instability

Initial :





About 0.5 sec of physical time

Grid 400x400, about 1.5 day of computation (sequential)

Concluding remarks

- Innovative numerical Eulerian method involving :
  - a Lagrange-Remap finite volume method
  - an anti-diffusive approach and the void fraction to keep a thin interface between the two fluids
- The test cases show a good agreement between XP and other codes (dam break, sloshing events)

- Ongoing works : XP + num of water wave wall impact (Luc Lenain, Ken Melville, U. Delaware San Diego, Frédéric Dias, U. College Dublin).
- Need to add : physical viscosity, surface tension for further investigation and validation
- GPU parallel computation

Some videos ...

Papers & videos

- A. Bernard-Champmartin, F. De Vuyst, « A low diffusive Lagrange-Remap scheme for the simulation of violent air-water free-surface flows », under progress, to submit to J. Comput. Phys. (2012)

- A. Bernard-Champmartin, F. De Vuyst, A low diffusive Lagrange-remap scheme for the simulation of violent air-water free-surface flows. Applications to dam-break and sloshing events, in preparation.

- Videos :

  http://www.youtube.com/user/floriandevuyst/videos

Thank you for your attention